UNIT-4 ASSIGNMENT PROBLEM.

The assignment problem is a special case of the transportation problem in which the objective is to assign a number of origins to the equal no. of destinations at a minimum cost (or) maximum profit.

Mathematical formulation of assignment problem.

		Activity				Available
		A_1	A_2	$\cdots$	A_n	
Resource	R_1	C_{11}	C_{12}	$\cdots$	C_{1n}	1
	R_2	C_{21}	C_{22}	$\cdots$	C_{2n}	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	R_n	C_{n1}	C_{n2}	$\cdots$	C_{nn}	1
	Required	1	1	$\cdots$	1	

Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to the constraints :

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \& \quad \sum_{j=1}^{n} x_{ij} = 1 \ ;$$

$$x_{ij} = 0 \text{ (or) } 1 \quad \forall \ i = 1, 2, \ldots, n \ \& \ j = 1, 2, \ldots, n$$

The Assignment Method (or) Hungarian Method:

Step 1: Determine the cost table from the given problem

(i) If the no. of sources is equal to the no. of destinations, go to step 3.

(ii) If the no. of sources is not equal to the no. of destinations, go to step 2.

Step 2: Add a dummy source (or) dummy destination so that the cost table becomes a square matrix. The cost entries of dummy source / destinations are always zero.

Step 3: Locate the smallest element in each row of the given cost matrix and then subtract the same from each element of that row.

Step 4: In the reduced matrix obtained in step 3 locate the smallest element of each column and then subtract the same from each element of that column. Each column and row now have atleast one zero

Step5: In the modified matrix obtained in step 4, search for an optimal assignment as follows:

(a) Examine the rows successively until a row with a single zero is found. Enrectangle this zero (□) and cross off (x)-all other zeros in its column. continue in this manner until all the rows have been take care of.

(b) Repeat the procedure for each column of the reduced matrix.

(c) If a row and/or column has two or more zeros and one cannot be chosen by inspection then assign arbitrary any one of these zeros and cross off all other zeros of that row/column.

(d) Repeat (a) through (c) above successively until the chain of assignning (□) or cross (x) ends.

Step6: If the number of assignments (□) is equal to n (the order of the cost matrix), an optimum solution is reached.

If the no. of assignments is less than n (the order of the matrix), go to the next step.

Step7: Draw the minimum no. of horizontal and/or vertical lines to cover all the zeros of the reduced matrix. This can be conveniently done by using a simple procedure.

(a) Mark (✓) rows that do not have any assigned zero.

(b) Mark (✓) columns that have zeros in the marked rows.

(c) Mark (✓) rows that have assigned zeros in the marked columns.

(d) Repeat (b) and (c) above until the chain of marking is completed.

(e) Draw lines through all the unmarked rows and marked columns. This gives us the desired minimum no. of lines.

Step8: Develop the new revised cost matrix as follows:

(a) Find the smallest element of the reduced matrix not covered by any of the lines.

(b) Subtract this element from all the uncovered elements and add the same to all the elements lying at the intersection of any two lines.

Step 9: Go to step 6 and repeat the procedure until an optimum solution is attained.

Problems:

1101. Soln:

	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

Subtracting the smallest element of each row

Row reduction

7	15	6	0
0	15	1	13
23	4	3	0
9	16	14	0

Subtracting the smallest element of each column

Column reduction

7	11	5	$\boxed{0}$
$\boxed{0}$	11	0	13
23	$\boxed{0}$	2	0
9	12	13	0

zeros 3 ≠ 4 Tasks

∴ P ≠ n.

The smallest no. not covered by the lines.

Subtract the element from uncovered elements

Adding the element from intersect the line.

	E	F	G	H
A	2	6	$\boxed{0}$	~~0~~
B	$\boxed{0}$	11	~~0~~	18
C	23	$\boxed{0}$	2	5
D	4	7	8	$\boxed{0}$

∴ P = n.

Ans: A → G, B → E, C → F, D → H.

The min. total time for this assignment = 17 + 13 + 19 + 10

1102 A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine & no - - - -

cost matrix $\begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$

Soln·

Given

	M_1	M_2	M_3
J_1	8	7	6
J_2	5	7	8
J_3	6	8	7

Row reduction

	M_1	M_2	M_3
J_1	2	1	0
J_2	0	2	3
J_3	0	2	1

column reduction

	M_1	M_2	M_3
J_1	2	0	0
J_2	0	1	3
J_3	0	1	1

$\therefore$ $p < n$

The minimum no. of lines (2) < the order of the cost number

	M_1	M_2	M_3
J_1	3	0	0
J_2	0	0	2
J_3	0	0	0

$\therefore p = n$

The optimum assignment is

	M_1	M_2	M_3
J_1		0	$\cancel{0}$
J_2	0	$\cancel{0}$	
J_3	$\cancel{0}$	$\cancel{0}$	0

$\therefore$ The optimum assignment is

Job 1 → Machine 2,

Job 2 → Machine 1,

Job 3 → Machine 3.

The total minimum cost $= 7 + 5 + 7 = 19$

1107.

Surplus Cities		Defit cities a	b	c	d	e
	A	85	75	65	125	75
	B	90	78	66	132	78
	C	75	66	57	114	69
	D	80	72	60	120	72
	E	76	64	50	112	68

Soln.

Row Reduction

20	10	0	60	10
24	12	0	66	12
18	9	0	57	12
20	12	0	60	12
20	8	0	56	12

Colum Reduction :

2	2	0	4	0
6	4	0	10	2
0	1	0	1	2
2	4	0	4	2
2	0	0	0	2

Draw the minimum lines to cover all the 0's

2	2	0	4	0
6	4	0	10	2
0	1	0	1	2
2	4	0	4	2
2	0	0	0	2

: $p < n \rightarrow 4 < 5$

Select the smallest element we add a smallest element to the covered and Substract the uncovered

2	2	2	4	0
4	2	0	8	0
0	1	2	0	2
0	2	0	2	0
2	0	2	0	2

$P = n$

	a	b	c	d	e
A					0
B			0		$\cancel{0}$
C	$\cancel{0}$			0	
D	0		$\cancel{0}$		$\cancel{0}$
E		0		$\cancel{0}$	

The optimum assignment is

$A \to e$, $B \to c$, $C \to d$, $D \to a$, $E \to b$

The total minimum cost $= 75 + 66 + 114 + 80 + 64$

$= 399$ km

110h

Task	Men 1	2	3
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

Soln.

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

Row Reduction

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

Column Reduction

0	6	9	0
4	7	0	0
26	0	9	0
9	10	14	0

Draw the ~~...~~ minimum lines to cover all the 0's

0	6	9	0
4	7	0	0
26	0	9	0
9	10	14	0

The optimum assignment is

I → 1, II → 3, III → 2, IV → 4

The minimum time is = 9 + 6 + 20 + 0

= 35 hours

1105 Soln

Row reduction

0	8	7	5
11	0	10	34
2	3	5	0
0	11	9	5

column reduction

0	8	2	5
11	0	5	4
2	3	0	0
0	11	4	5

Draw the minimum no of lines to cover all the 0's

0	8	2	5
11	0	5	4
2	3	0	0
0	11	4	5

Since $P < n$

Here 2 is the smallest element in the uncrossed cell. We add 2 to the crossed and subtract the uncrossed cell, we get

0	6	0	3
13	0	5	4
3	3	0	0
0	9	2	8

p = n

A	0	6	$\boxed{0}$	3
B	13	$\boxed{0}$	5	4
C	3	8	0	$\boxed{0}$
D	$\boxed{0}$	9	2	3

The optimal assignment is

A → dynamic programe   B → queuing theory

C → Regression analysis   D → linear programme

Total course preparation time } = 9 + 4 + 11 + 4 = 28

1106 **Row reduction**

	A	B	C	D
1	3	1	0	6
2	5	7	0	4
3	2	0	1	3
4	0	2	2	3

Colum reduction

	A	B	C	D
1	3	1	0	3
2	5	7	0	1
3	2	0	1	0
4	0	2	2	0

Draw the minimum no of lines to cover all the 0's

3	1	0	3
5	7	0	1
2	0	1	0
0	2	2	0

Since p < n there 1 is the smallest element We add 1 from intersected cells and subtract 1 from uncrossed cell

$$\begin{matrix} 2 & 0 & 0 & 2 \\ 3 & 5 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 3 & 0 \end{matrix}$$

$\therefore$ $p = n$

The optimal assignment is

$$\begin{matrix} 2 & \boxed{0} & 0 & 2 \\ 3 & 5 & \boxed{0} & 0 \\ 2 & 0 & 2 & \boxed{0} \\ \boxed{0} & 2 & 3 & 0 \end{matrix}$$

Man 1 → Job 2, Man 2 → Job 3, Man 3 → Job

Man 4 → Job 1

$\therefore$ Minimum man hours $= 3+2+7+5 = 17$

1107 a) Row reduction

$$\begin{matrix} 0 & 3 & 5 & 2 \\ 2 & 0 & 3 & 2 \\ 0 & 1 & 6 & 2 \\ 3 & 4 & 3 & 0 \end{matrix}$$

Column reduction

$$\begin{matrix} 0 & 3 & 2 & 2 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 3 & 2 \\ 3 & 4 & 0 & 0 \end{matrix}$$

Draw the minimum no. of lines to cover all the 0's

$$\begin{matrix} 0 & 3 & 2 & 2 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 3 & 2 \\ 3 & 4 & 0 & 0 \end{matrix}$$

Since $p < 4$ here 1 is the smallest element in the uncrossed cell we add 1 to the crossed cell & subtract the uncrossed cell

0	2	1	1
3	0	0	2
0	0	2	1
3	4	0	0

$\boxed{0}$	2	1	1
3	0	$\boxed{0}$	2
0	$\boxed{0}$	2	1
3	4	0	$\boxed{0}$

The optimal assignment is

I → A , II → C , III → B , IV → D

Total minimum cost = 1 + 10 + 5 + 5 = 21

(b) Row reduction

0	15	5	10
10	25	0	10
23	8	0	12
0	8	7	3

column reduction

0	7	5	7
10	17	0	7
23	0	0	9
0	0	7	0

Draw the minimum no of lines to cover all the 0's

0	7	5	7
10	17	0	7
23	0	0	9
0	0	7	0

$p = n$

$\boxed{0}$	7	5	7
10	17	$\boxed{0}$	7
23	$\boxed{0}$	0	9
0	0	7	$\boxed{0}$

The optimal assignment is

1 → A , 2 → C , 3 → B , 4 → D

The minimum cost = 10 + 5 + 20 + 20 = 55

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c. Row reduction

$$\begin{matrix} 0 & 2 & 9 & 1 \\ 0 & 5 & 2 & 3 \\ 1 & 3 & 2 & 0 \\ 0 & 7 & 3 & 1 \end{matrix}$$

Column reduction

$$\begin{matrix} 0 & 0 & 7 & 1 \\ 0 & 3 & 0 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 5 & 1 & 1 \end{matrix}$$

Draw the minimum no of lines to cover all

$$\begin{matrix} 0 & 0 & 7 & 1 \\ 0 & 3 & 0 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 5 & 1 & 1 \end{matrix}$$

P = n

$$\begin{matrix} 0 & \boxed{0} & 7 & \\ 0 & 3 & \boxed{0} & 3 \\ 1 & 1 & 0 & 0 \\ \boxed{0} & 5 & 1 & 1 \end{matrix}$$

The optimal assignment is

$A \to 2$, $B \to 3$, $C \to 4$, $D \to 1$

Total minimum cost $= 10 + 7 + 11 + 8 = 36$

d. Row reduction

$$\begin{matrix} 3 & 6 & 1 & 0 \\ 5 & 2 & 0 & 3 \\ 0 & 6 & 8 & 6 \\ 4 & 2 & 5 & 0 \end{matrix}$$

Column reduction

$$\begin{matrix} 3 & 4 & 1 & 0 \\ 5 & 0 & 0 & 3 \\ 0 & 4 & 8 & 6 \\ 4 & 0 & 5 & 0 \end{matrix}$$

Draw the minimum no of lines to cover all

$$\begin{matrix} 3 & 4 & 1 & 0 \\ 5 & 0 & 0 & 3 \\ 0 & 4 & 8 & 6 \\ 4 & 0 & 5 & 0 \end{matrix}$$

P = n

$$\begin{matrix} 3 & 4 & 1 & \boxed{0} \\ 5 & 0 & \boxed{0} & 3 \\ \boxed{0} & 4 & 8 & 6 \\ 4 & \boxed{0} & 5 & 0 \end{matrix}$$

The optimal assignment is

$J_1 \to M_4$, $J_2 \to M_3$, $J_3 \to M_1$, $J_4 \to M_2$

Total minimum cost $= 2 + 5 + 4 + 6 = 17$

1108 Row reduction

$$\begin{matrix} 3 & 2 & 0 & 4 \\ 1 & 0 & 3 & 2 \\ 0 & 3 & 1 & 2 \\ 0 & 11 & 23 & 32 \end{matrix}$$

Column reduction

$$\begin{matrix} 3 & 2 & 0 & 2 \\ 1 & 0 & 3 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 11 & 23 & 30 \end{matrix}$$

Draw the minimum no. of lines to cover all 0's

$$\begin{matrix} 3 & 2 & 0 & 2 \\ 1 & 0 & 3 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 11 & 23 & 30 \end{matrix}$$

P = n

$$\begin{matrix} 3 & 2 & \boxed{0} & 2 \\ 1 & \boxed{0} & 3 & 0 \\ 0 & 3 & 1 & \boxed{0} \\ \boxed{0} & 11 & 23 & 30 \end{matrix}$$

The optimal assignment is

A → III, B → II, C → IV, D → I

Total minimum cost : 12 + 22 + 33 + 21 = 88

1109 Row reduction

$$\begin{matrix} 2 & 0 & 1 & 4 \\ 0 & 1 & 4 & 2 \\ 8 & 2 & 0 & 1 \\ 1 & 3 & 0 & 2 \end{matrix}$$

Column reduction

$$\begin{matrix} 2 & 0 & 1 & 3 \\ 0 & 1 & 4 & 1 \\ 8 & 2 & 0 & 0 \\ 1 & 3 & 0 & 1 \end{matrix}$$

Draw minimum lines to cover all 0's

$$\begin{matrix} 2 & 0 & 1 & 3 \\ 0 & 1 & 4 & 1 \\ 8 & 2 & 0 & 0 \\ 1 & 3 & 0 & 1 \end{matrix}$$

P = n

$$\begin{matrix} 2 & \boxed{0} & 1 & 3 \\ \boxed{0} & 1 & 4 & 1 \\ 8 & 2 & 0 & \boxed{0} \\ 1 & 3 & \boxed{0} & 1 \end{matrix}$$

The optimal assignment is

Job 1 → Operator 2, Job 2 → Operator 1, Job 3 → Operator 4

1110 Row reduction

$$\begin{matrix} 1 & 0 & 2 & 3 \\ 0 & 2 & 2 & 1 \\ 3 & 1 & 0 & 4 \\ 4 & 0 & 2 & 1 \end{matrix}$$

Column reduction

$$\begin{matrix} 1 & 0 & 2 & 2 \\ 0 & 2 & 2 & 0 \\ 3 & 1 & 0 & 3 \\ 4 & 0 & 2 & 0 \end{matrix}$$

Draw the minimum no. of lines to cover all 0's

P = n

$$\begin{matrix} 1 & 0 & 2 & 2 \\ 0 & 2 & 2 & 0 \\ 3 & 1 & 0 & 3 \\ 4 & 0 & 2 & 0 \end{matrix}$$

$$\begin{matrix} 1 & \boxed{0} & 2 & 2 \\ \boxed{0} & 2 & 2 & 0 \\ 3 & 1 & \boxed{0} & 3 \\ 4 & 0 & 2 & \boxed{0} \end{matrix}$$

The optimal assignment is

A → X, B → W, C → Y, D → Z

Total minimum cost = 7 + 7 + 7 + 7 = 28

1111. Row reduction

$$\begin{matrix} 7 & 0 & 7 & 4 & 4 \\ 0 & 4 & 2 & 6 & 4 \\ 11 & 16 & 0 & 7 & 8 \\ 13 & 1 & 0 & 5 & 2 \\ 14 & 4 & 0 & 10 & 10 \end{matrix}$$

Column reduction

$$\begin{matrix} 7 & 0 & 7 & 0 & 2 \\ 0 & 4 & 2 & 2 & 2 \\ 11 & 16 & 0 & 3 & 6 \\ 13 & 1 & 0 & 1 & 0 \\ 14 & 4 & 0 & 6 & 8 \end{matrix}$$

Draw the minimum no of lines to cover all

$$\begin{matrix} 7 & 0 & 7 & 0 & 2 \\ 0 & 4 & 2 & 2 & 2 \\ 11 & 16 & 0 & 3 & 6 \\ 13 & 1 & 0 & 1 & 0 \\ 14 & 4 & 0 & 6 & 8 \end{matrix}$$

Here 2 is the smallest element in the uncrossed cell We add 2 to the crossed cell & subtract the uncrossed cell

9	0	9	0	2
0	2	2	0	0
11	14	0	1	4
15	1	2	1	0
14	2	0	4	6

$p < n$

Here 1 is the smallest element in the uncrossed cell We add 1 to the crossed cell & subtract the uncrossed cell

9	0	10	0	2
0	2	3	0	0
10	13	0	0	3
15	1	3	1	0
13	1	0	3	5

$p = n$

9	$\boxed{0}$	10	0	2
$\boxed{0}$	2	3	0	0
10	13	0	$\boxed{0}$	3
15	1	3	1	$\boxed{0}$
13	1	$\boxed{0}$	3	5

The optimal assignment is

Operator 1 → Machine 2. Operator 2 → Machine 1.

Operator 3 → Machine 4. Operator 4 → Machine 5.

Operator 5 → Machine 3

Total minimum cost = 3 + 5 + 9 + 4 + 2 = 23

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1112 Row reduction

$$\begin{matrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{matrix}$$

Column reduction

$$\begin{matrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{matrix}$$

Draw the minimum no. of lines cover all 0's

$$\begin{matrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{matrix}$$

$$\begin{matrix} 7 & 3 & 0 & 5 & \boxed{0} \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & 0 & 3 \\ 4 & \boxed{0} & 2 & 4 & 0 \end{matrix}$$

$P = n$

The optimal assignment is

Person A → Job 5, Person B → Job 1, Person C → Job 4

Person D → Job 3, Person E → Job 2

Total minimum cost = 1 + 0 + 2 + 1 + 5 = 9

116 Row reduction

2	4	17	4	0
0	22	20	20	3
9	0	3	6	5
9	0	1	3	0
0	3	6	7	9

Column reduction

2	4	16	2	[illegible]
0	22	19	19	[illegible]
9	0	2	4	[illegible]
2	0	0	2	[illegible]
0	3	4	2	[illegible]

$p < n$

9	4	14	1	0
0	22	11	17	2
9	0	0	0	5
4	0	0	0	2
0	3	2	0	9

$p = n$

	O_1	O_2	O_3	O_4	O_5
J_1					$\boxed{0}$
J_2	$\boxed{0}$				
J_3		$\boxed{0}$	0		
J_4			$\boxed{0}$		
J_5	0			$\boxed{0}$	

The optimum assignment is $J_1 \to O_5$, $J_2 \to O_1$, J_3 $J_4 \to O_3$ & $J_5 \to O_4$

The minimum cost $= 18 + 4 + 14 + 16 + 19 = 71$

III 8 Row reduction

16	47	14	27	0	26
0	7	27	43	29	28
0	12	33	24	5	5
8	13	11	15	0	6
3	14	13	0	4	7
52	10	10	30	21	0

Column reduction

16	40	4	27	0	26
0	0	17	43	59	28
0	5	23	24	5	5
8	6	1	15	0	6
3	7	3	0	4	7
52	3	0	30	21	0

$P < n$

The smallest element is 1

16	40	3	27	0	25
0	0	16	43	59	27
0	5	22	24	5	4
8	6	0	15	0	5
3	7	2	0	4	6
52	4	0	31	22	0

$P = n$

	7	8	9	10	11	12
1					$\boxed{0}$	
2	0	$\boxed{0}$				
3	$\boxed{0}$					
4			$\boxed{0}$			
5				$\boxed{0}$		
6			0			$\boxed{0}$

The optimum assignment is $1 \to 11$, $2 \to 8$, $3 \to 7$, $4 \to 9$, $5 \to 10$, $6 \to 12$

1120 Row reduction

0	6	10	14
0	5	9	11
0	5	9	12
0	0	0	0

column reduction

0	6	10	14
0	5	9	11
0	5	9	12
0	0	0	0

$p < n \Rightarrow 2 < 4$

The smallest element is 5. We add 5 to crossed cell We subtract 5 to uncrossed cell

$$\begin{matrix} 0 & 1 & 5 & 9 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 4 & 7 \\ 5 & 0 & 0 & 0 \end{matrix}$$

$P < n \Rightarrow 3 < 4$

Here 4 is the smallest element We add 4 to the crossed cell subtract 5 to the uncrossed cell

$$\begin{matrix} 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 9 & 4 & 0 & 0 \end{matrix} \qquad \begin{matrix} \boxed{0} & 1 & 1 & 5 \\ 0 & \boxed{0} & 0 & 2 \\ 0 & 0 & \boxed{0} & 3 \\ 9 & 4 & 0 & \boxed{0} \end{matrix}$$

$P = n$

The optimal assignment is

Job 1 → Machine A , Job 2 → Machine B,

Job 3 → Machine C, Job 4 → Machine D

Total minimum cost = 18 + 13 + 19 + 0 = 50

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11.21. Soln:

We add the dummy column

	a	b	c	d	e
A	50	40	60	20	0
B	40	30	40	30	0
C	60	20	30	20	0
D	30	30	20	30	0
E	10	20	10	30	0

Row reduction:

50	40	60	20	0
40	30	40	30	0
60	20	30	20	0
30	30	20	30	0
10	20	10	30	0

Column reduction

40	20	50	0	0
30	10	30	10	0
50	0	20	0	0
20	10	10	20	0
0	0	0	10	0

$P < n$

Choose the smallest element

30	10	40	0	0
20	0	20	10	0
50	0	20	10	10
10	0	0	20	0
0	0	0	20	10

$P = n$

	a	b	c	d	e
A				$\boxed{0}$	0
B		0			$\boxed{0}$
C		$\boxed{0}$			
D	0	0	$\boxed{0}$		0
E	$\boxed{0}$	0	0		

The optimal assignment is A→d, B→e,

W20 C→b, D→c, E→a

The minimum cost = 30 + 0 + 30 + 20 + 10 = 70

1122 Soln

We add the dummy column

	A	B	C	D	E
1	30	27	31	39	0
2	28	18	28	37	0
3	33	17	29	41	0
4	27	18	30	43	0
5	40	20	27	36	0

Row reduction

30	27	31	39	0
28	18	28	37	0
33	17	29	41	0
27	18	30	43	0
40	20	27	36	0

Column reduction

3	10	4	3	0
1	1	1	1	0
6	0	2	5	0
0	1	2	7	0
13	3	0	0	0

The least element is 1

2	10	3	2	0
0	1	0	0	0
5	0	1	4	0
0	2	3	7	1
13	4	0	0	1

P = n

	A	B	C	D	E
1					$\boxed{0}$
2	0		$\boxed{0}$	0	0
3		$\boxed{0}$			0
4	$\boxed{0}$				
5			0	$\boxed{0}$	

The optimal assignment is 1→E, 2→C, 3

4→A, 5→D

The minimum cost = 0 + 28 + 17 + 27 + 36 = 108

11.23

	I	II	III	IV	V	VI
A	73	91	87	82	78	80
B	81	85	69	76	74	85
C	75	72	83	84	72	91
D	93	96	86	91	83	82
E	90	91	79	89	69	76
F	0	0	0	0	0	0

Row reduction

0	18	14	9	5	7
12	16	0	7	5	16
3	0	11	12	6	19
11	14	4	9	1	0
21	22	10	20	0	7
0	0	0	0	0	0

column reduction.

0	19	14	9	5	7
12	16	0	7	5	16
3	0	11	12	6	19
11	14	4	9	1	0
21	22	10	20	0	7
0	0	0	0	0	0

P = n

The least element is 7

0	18	14	2	5	7
12	16	0	0	5	16
3	0	11	5	6	19
11	14	4	2	1	0
21	22	10	13	0	7
7	7	7	0	7	7

P = n

	I	II	III	IV	V	VI
A	$\boxed{0}$					
B			$\boxed{0}$			
C		$\boxed{0}$				
D						$\boxed{0}$
E					$\boxed{0}$	
F				$\boxed{0}$		

The optimal assignment is A → I, B → III, C → II, D → VI, E → V, F → IV

The minimum cost = 73 + 69 + 72 + 82 + 69 + 0 = 365